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Cynk, Sławomir (PL-JAGL); **van Straten, Duco** (D-MNZ)
Infinitesimal deformations of double covers of smooth algebraic varieties. (English summary)
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This paper studies the tangent space to the deformation space of double covers of smooth varieties, with an aim to calculating Hodge numbers of Calabi-Yau threefolds constructed as resolutions of double covers.

Let $X \to Y$ be such a double cover. The authors identify two subspaces of interest in $H^1(\Theta_X)$, the tangent space to the deformation space of X. These are $T^1_{X\to Y}$, the infinitesimal deformations of X which are double covers of deformations of Y, and $T^1_{X/Y}$, the deformations of X which remain double covers of Y. Clearly $T^1_{X/Y} \subseteq T^1_{X\to Y}$. The authors identify $T^1_{X\to Y}$ with $H^1(\Theta_Y(\log D))$, where D is the branch locus of the double cover, and $T^1_{X/Y}$ with $\operatorname{coker}(H^0(\Theta_Y) \to H^0(\mathcal{N}_{D/Y}))$. If D is singular, one can construct a resolution of X by using an embedded resolution of D inside Y; this will yield $D^* \subseteq \tilde{Y}$ and a double cover $\tilde{X} \to \tilde{Y}$. $H^1(\Theta_{\tilde{Y}}(\log D^*))$ is then isomorphic to the space of deformations of the pair $D \subset Y$ which have simultaneous resolution, i.e. equisingular deformations. The main result of the paper then gives an explicit way of calculating this cohomology group. For covers of projective space, this can be expressed as the degree d part (where d is the degree of D) of what the authors term the equisingular ideal of D. The paper ends with some Singular code for some sample calculations.

Reviewed by *Mark Gross*

References

1. C. Birkenhake, H. Lange, and D. van Straten, Abelian surfaces of type $(1,4)$, Math. Ann. **285,** No. 4, 625–646 (1989). MR1027763 (91b:14042)
2. C. H. Clemens, Double solids, Adv. Math. **47,** 107–230 (1983). MR0690465 (85e:14058)
3. S. Cynk, Double coverings of octic arrangements with isolated singularities, Adv. Theor. Math. Phys. **3,** 217–225 (1999). MR1736791 (2000m:14043)
4. S. Cynk and T. Szemberg, Double covers and Calabi–Yau varieties, Banach Center Publ. **44,** 93–101 (1998). MR1677335 (2000b:14047)
5. H. Esnault and E. Viehweg, Lectures on vanishing theorems, DMV Seminar Vol. 20 (Birkhäuser, Basel, 1992). MR1193913 (94a:14017)
6. P. Griffiths and J. Harris, Principles of Algebraic Geometry (Wiley and Sons, New York, 1978). MR0507725 (80b:14001)
7. G.-M. Greuel, G. Pfister, and H. Schönemann, Singular 2.0. A Computer Algebra System for Polynomial Computations, Centre for Computer Algebra, University of Kaiserslautern (2001). http://www.singular.uni-kl.de

8. R. Hartshorne, Algebraic Geometry (Springer, New York–Heidelberg, 1977). MR0463157 (57 #3116)
9. Y. Kawamata, An equisingular deformation theory via embeded resolution of singularities, Publ. Res. Inst. Math. Sci. **16,** 233–244 (1980). MR0574034 (81g:32010)
10. B. Szendröi, Calabi–Yau threefolds with a curve of singularities and counterexamples to the Torelli problem, Internat. J. Math. **11,** No. 3, 449–459 (2000). MR1769616 (2002a:14045)
11. B. Szendröi, Calabi–Yau threefolds with a curve of singularities and counterexamples to the Torelli problem. II, Math. Proc. Cambridge Philos. Soc. **129,** No. 2, 193–204 (2000). MR1765909 (2002a:14046)
12. J. M. Wahl, Equisingular deformations of plane algebroid curves, Trans. Amer. Math. Soc. **193,** 143–170 (1974). MR0419439 (54 #7460)
13. P. M. H. Wilson, The Kähler cone on Calabi–Yau threefolds, Invent. Math. **107,** No. 3, 561–583 (1992). MR1150602 (93a:14037)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.